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PERFORMANCE ANALYSIS OF FOUR-POINT EGAOR ITERATIVE METHOD APPLIED TO POISSON IMAGE BLENDING PROBLEM

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ABSTRACT Poisson image blending is one of the useful editing tools in image processing to generate a desirable image which is impossible to acquire. The key to this solution is to obtain the unique solution of Poisson equation. Thus, the motivation of this paper is to examine the effectiveness of 4-EGAOR iterative method to solve the linear system generated from the Poisson image blending problem. To evaluate its effectiveness, the formulation and implementation of 4-EGAOR, SOR and AOR iterative methods are demonstrated. The numerical results revealed that 4-EGAOR iterative method improved the computational time taken and reduced the number of iterations used. In fact, the new images generated by the proposed block iterative method give a satisfactory visual effect.

Keywords Poisson equation, Poisson image blending, finite difference method, 4-EGAOR iteration, five-point Laplacian operator

ABSTRAK Penyuntingan imej Poisson merupakan salah satu alat pengeditan yang berguna dalam dunia pemprosesan imej untuk menghasilkan suatu imej yang diinginkan di mana ia adalah mustahil untuk diperoleh. Kunci penyelesaian permasalahan ini ialah mendapatkan penyelesaian unik persamaan Poisson. Oleh itu, motivasi kajian ini adalah untuk menganalisis kecekapan kaedah lelaran 4-EGAOR untuk menyelesaikan sistem linear yang terjana daripada permasalahan penyuntingan imej Poisson. Untuk menilai kecekapannya, perumusan dan pelaksanaan kaedah 4-EGAOR, SOR dan AOR telah ditunjukkan. Dapatan kajian menunjukkan bahawa kaedah lelaran 4-EGAOR telah mengurangkan masa dan bilangan lelaran untuk menyelesaikan permasalahan imej Poisson berbanding dengan kaedah lelaran SOR dan AOR. Kualiti imej yang dihasilkan oleh semua kaedah lelaran yang dicadangkan tiada mempunyai perbezaan yang ketara.

INTRODUCTION

When the photographs are represented in digital form, it is known as digital images and when it is being processed by a digital computer, it is regarded as digital image processing. Digital image processing is present in many aspects of our daily life, with broad applications such as medical imaging, geographical mapping, remote sensing and observation of the earth resources. The origins of digital image processing can be traced back to Gonzalez and Woods (2001).

Nevertheless, in this research study, we focus on examining the Poisson image blending problem, which is also known as Poisson Image Editing (PIE). Solving image blending problem by imposing Poisson partial differential equation with Dirichlet boundary condition was invented by Pérez *et al*. (2003). Generally, PIE is defined as inserting a desired region from source image into the target image seamlessly by solving the Poisson equation. However, the Dirichlet boundary condition often caused the issue of color inconsistency. An extra inner constrained boundary condition was added to the edited region by Qin *et al*. (2008), therefore the Laplacian values was enlarged in order to overcome the issue.

In addition, Morel *et al*. (2012) reviewed that PIE is a lengthy process which involved a longer composition time. On account of this, Fourier method was implemented. This is a fast and efficient non-iterative type of solver. More recently, Hussain and Kamel (2015) proposed the method of image pyramid and divide-and-conquer to solve the Poisson equation more efficiently. The Poisson equation was solved for three pyramid levels in this method. Besides, modified Poisson blending (MPB) solver had proposed by Afifi and Hussain (2015) in order to overcome the issues of bleeding artifacts and colour bleeding. To overcome the bleeding artifacts problem which occurred in the traditional methods, both the boundary pixels on the source and destination images were considered.

There are plenty of techniques being applied in Poisson image blending problem and their efficiency in resolving this problem are high. Generally, the linear system generated from the Poisson equation can be solved by direct and iterative methods. In this paper, we discretized the Poisson equation based on finite difference approach and then solved the linear system by using iterative method. Only few researches employed iterative method in Poisson image blending problem, Hussain and Kamel (2015), Eng *et al*. (2017a), Eng *et al*. (2017b), Eng *et al*. (2018a) and Eng *et al*. (2018b). Thus, this paper is examining the efficiency of the Four-Point Explicit Group Accelerated Over Relaxation (4-EGAOR) when applied to Poisson image blending problem as none of the researches have employed this technique in the problem.

In the meantime, Martino *et al*. (2016) evaluated that finite difference approach is well performed in most of the Poisson image blending problem. Thus, three selected iterative methods based on this approach are proposed in this study: 4-EGAOR and point iterative methods, Successive Over Relaxation (SOR) and Accelerated Over Relaxation (AOR). Besides solving image blending problem, 4-EGAOR, SOR and AOR iterative methods and its variants had been used to solve other problems as well, for example in Akhir *et al*. (2011), Dahalan *et al*. (2017) and Saudi and Sulaiman (2017). Moreover, the Laplace's equation which is generated from Poisson equation by simply assigning the value of zero to the right hand side of Poisson equation, is also applied in path planning problem (Saudi & Sulaiman, 2012; Saudi & Sulaiman, 2016).

MODEL DESCRIPTION

An image is a two-dimensional array of finite number of pixels where each of the pixels represent its own intensity value or brightness. The coordinate system of an image is the rotated conventional two-dimensional Cartesian coordinate system by 90º in clockwise direction. Besides, a colour image is formed by various mixtures of three primary colours, Red, Green and Blue (RGB) and the size or resolution of an image is determined by the number of rows and columns in the image. Referring to Figure 1, the image is partitioned into many smaller square parts where each of them is representing a pixel.

Figure 1. Illustration of an image partitioned into 28×18 small square blocks.

For convenience purpose, the image shown in Figure 1 will be simplified by considering the location of the pixels from the solution domain, as shown in Figure 2.

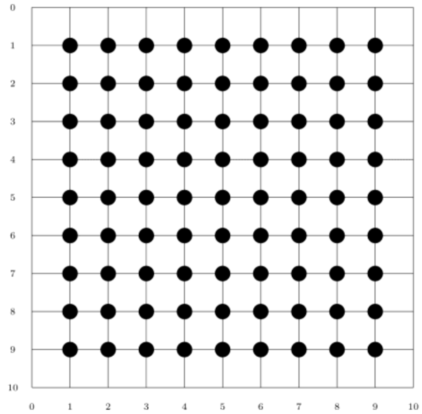


Figure 2. Finite grid network of pixels for n=10.

PIE is a type of gradient domain image editing method where it manipulates the gradients instead of the pixels of the image. The Poisson blending process begins by selecting the desired region O with ∂O defined as its boundary from a source image

g. Then, extract the desired region and blend into a destination image f* to generate a new output image f. As stated by Pérez *et al*. (2003), a guidance vector field v needs to be generated first from the source image. After that, a new set of intensity values f in the desired region which will minimize the difference between the gradient of the new image and the vector field is computed. The minimization problem is defined as,

$$\min_{\mathrm{f}} \iint_{\mathrm{O}} |\nabla\mathrm{f}\text{-}\mathrm{v}|^2 \text{ with } \mathrm{f}|_{\partial\mathrm{O}} = \mathrm{f}^*|_{\partial\mathrm{O}} \quad (1)$$

where the intensity values are set to be the same at the boundary to generate a seamless image. The intensity values are extracted from the image itself. It will be used as the initial values for the generated linear system constructed from the discretization Poisson equation via finite difference method. The details are discussed in the following section.

The solution of the minimization problem (1) is the unique solution of the Poisson equation with Dirichlet boundary condition,

$$\Delta f = \operatorname{div} \mathbf{v} \text{ at O with } f|_{\partial\mathrm{O}} = f^*|_{\partial\mathrm{O}} \quad (2)$$

To generate a new image seamlessly, the vector field v is a gradient field directly extracted from the source image and thus, Equation (2) can be rewritten as,

$$\Delta f = \Delta g \text{ at O with } f|_{\partial\mathrm{O}} = f^*|_{\partial\mathrm{O}} \quad (3)$$

The Δ in Equation (3) denotes the Laplacian operator.

DERIVATION AND IMPLEMENTATION OF THE PROPOSED METHOD

To implement the three proposed iterative methods, Poisson equation is first to be discretized based on finite difference approach via Laplacian operator. The standard two-dimensional Poisson equation is defined as,

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = g(x, y) \quad (4)$$

Equation (4) is then discretized via five-point Laplacian operator. From the discretization process of Equation (4), the approximation equation is derived as,

$$f_{i,j} \cong \frac{1}{4}\left[f_{i+1,j} + f_{i-1,j} + f_{i,j+1} + f_{i,j-1} - h^2 g_{i,j}\right] \quad (5)$$

where $h = \Delta x = \Delta y$. By adding a weighted parameter, ω to Equation (5), it can be redefined as,

$$f_{i,j}^{(k+1)} \cong \frac{\omega}{4}\left[f_{i+1,j}^{(k)} + f_{i-1,j}^{(k+1)} + f_{i,j+1}^{(k)} + f_{i,j-1}^{(k+1)} - h^2 g_{i,j}\right] + (1-\omega) f_{i,j}^{(k)} \qquad (6)$$

and by adding another accelerated parameter, r into Equation (6), it can also be redefined as,

$$\begin{aligned} f_{i,j}^{(k+1)} \cong \frac{r}{4}\left(f_{i-1,j}^{(k+1)} - f_{i-1,j}^{(k)} + f_{i,j-1}^{(k+1)} - f_{i,j-1}^{(k)}\right) + \\ \frac{\omega}{4}\left(f_{i-1,j}^{(k)} + f_{i+1,j}^{(k)} + f_{i,j-1}^{(k)} + f_{i,j+1}^{(k)} - h^2 g_{i,j}\right) + (1-\omega) f_{i,j}^{(k)} \end{aligned} \qquad (7)$$

Thus, Equation (6) and (7) is the formulation of SOR (Young, 1954) and AOR (Hadjidimos, 1978) iterative methods respectively with $1 \leq \omega, r < 2$ (Ali & Chong, 2007; Yousif & Martins, 2008).

On the other hand, to implement the 4-EGAOR iterative method, let us consider a group of four points as shown in Figure 3 as follows,

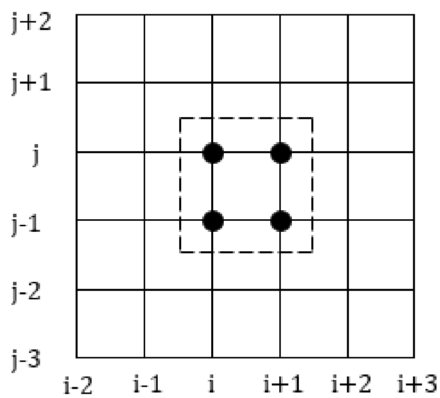


Figure 3. Four points block iterative method.

and the 4-EG iterative method (Saudi & Sulaiman, 2012; Evans, 1985) is defined as,

$$\begin{bmatrix} 4 & -1 & -1 & 0 \\ -1 & 4 & 0 & -1 \\ -1 & 0 & 4 & -1 \\ 0 & -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} f_{i,j} \\ f_{i+1,j} \\ f_{i,j+1} \\ f_{i+1,j+1} \end{bmatrix} = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix} \qquad (8)$$

where,

$S_1 = g_{i,j} + f_{i-1,j} + f_{i,j-1}$,
$S_2 = g_{i+1,j} + f_{i+2,j} + f_{i+1,j-1}$,
$S_3 = g_{i,j+1} + f_{i-1,j+1} + f_{i,j+2}$,
$S_4 = g_{i+1,j+1} + f_{i+2,j+1} + f_{i+1,j+2}$.

Then, the inverse of the coefficient matrix (8) is determined as follows,

$$\begin{bmatrix} f_{i,j} \\ f_{i+1,j} \\ f_{i,j+1} \\ f_{i+1,j+1} \end{bmatrix} = \frac{1}{24}\begin{bmatrix} 7 & 2 & 2 & 1 \\ 2 & 7 & 1 & 2 \\ 2 & 1 & 7 & 2 \\ 1 & 2 & 2 & 7 \end{bmatrix}\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix} \tag{9}$$

To implement 4-EGAOR (Martins *et al*., 2002) iterative method from Equation (9), let,

$$\begin{aligned} S_a &= \frac{1}{24}[S_1 + 2S_2 + 2S_3 + S_4], \\ S_b &= \frac{1}{24}[2S_1 + S_2 + S_3 + 2S_4]. \end{aligned} \tag{10}$$

$$\begin{aligned} a_1 &= f_{i-1,j}{}^{(k+1)} - f_{i-1,j}{}^{(k)}, \\ a_2 &= f_{i,j-1}{}^{(k+1)} - f_{i,j-1}{}^{(k)}, \\ a_3 &= f_{i+1,j-1}{}^{(k+1)} - f_{i+1,j-1}{}^{(k)}, \\ a_4 &= f_{i-1,j+1}{}^{(k+1)} - f_{i-1,j+1}{}^{(k)}. \end{aligned} \tag{11}$$

Then, Equations (10) and (11) are substituted into Equation (9) and two weighted parameters, ω and r with $1 \le \omega, r < 2$ are added. Thus, the 4-EGAOR iterative method can be implemented simultaneously by using the following formulation,

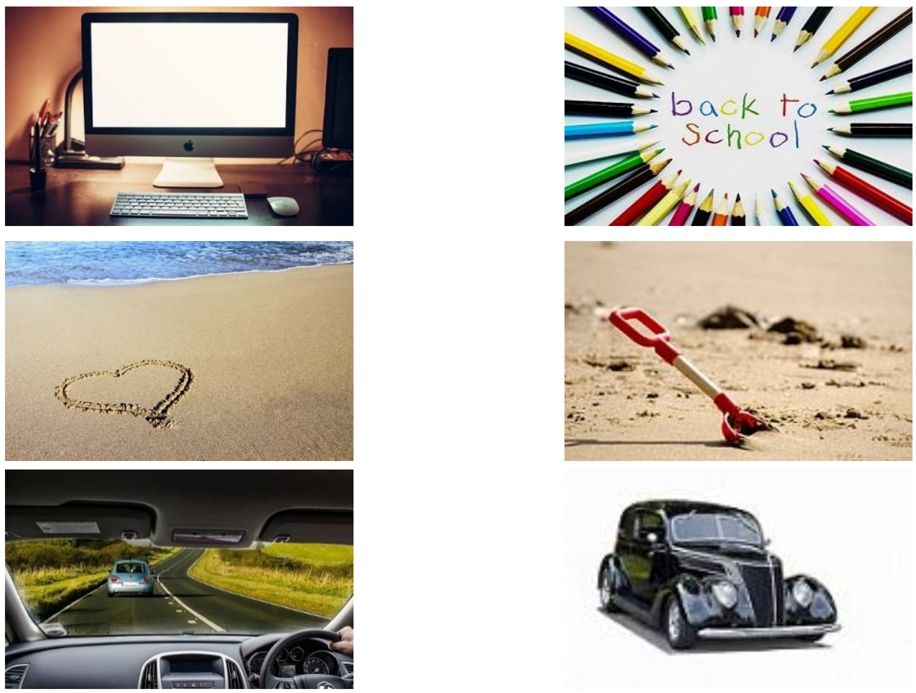
$$\begin{bmatrix} f_{i,j} \\ f_{i+1,j} \\ f_{i,j+1} \\ f_{i+1,j+1} \end{bmatrix}^{(k+1)} = \frac{r}{24}\begin{bmatrix} 7 & 7 & 2 & 2 \\ 2 & 2 & 7 & 1 \\ 2 & 2 & 1 & 7 \\ 1 & 1 & 2 & 2 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} + \frac{\omega}{4}\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix} + \omega\begin{bmatrix} S_a \\ S_b \\ S_b \\ S_a \end{bmatrix} + (1-\omega)\begin{bmatrix} f_{i,j} \\ f_{i+1,j} \\ f_{i,j+1} \\ f_{i+1,j+1} \end{bmatrix}^{(k)} \tag{12}$$

Algorithm 1: 4-EGAOR iterative method

i. Select desired region from source image.
ii. Map the desired region onto target image.
iii. Set the initial value $\underline{f}^{(0)} = 0$ and $\varepsilon = 1.0$.
iv. For all groups of four node points in the selected region, use Equation (12) to compute $f_{i,j}{}^{(k+1)}, f_{i+1,j}{}^{(k+1)}, f_{i,j+1}{}^{(k+1)}$ and $f_{i+1,j+1}{}^{(k+1)}$
v. For all ungroup node points in the selected region, use Equation (7) to compute.
vi. Examine the convergence. If it converges, go to step vii. Else, go to step iv.
vii. Display the new images obtained.

RESULTS AND DISCUSSION

In order to verify the potency of the proposed methods, three sets of images with various sizes are chosen. Each set of the images comprises of source and target images were taken from Public Domain Pictures.net refer to Figure 4. To evaluate the performance of the three selected iterative methods, two parameters are used, the number of iterations involved and the computational time taken to generate a new blended image. In this paper, we employed Root of Sum of Square (RSS) (Pérez *et al*., 2003), as stopping criterion with 1.0 threshold.



(i) (ii) (iii)

(a) (b)

Figure 4. (a) Target images. (b) Source images.

The performance evaluation of SOR, AOR and 4-EGAOR in terms of number of iterations used is presented in Figure 5 and the values of ω and r used are optimal. Referring to Figure 5, the number of iterations used by 4-EGAOR has reduced as compared to SOR and AOR iterative methods. It has decreased approximately 19.1 – 26.1% as compared to SOR iterative numerical result for comparison in the AOR iterative method. Meanwhile, the method and 7.3 – 21.4% as compared to composing time is presented in Figure 6. As compared to SOR iterative method, the composing time of 4-EGAOR is reduced by approximately 15.2 – 17.6% while compared to AOR iterative method, it has reduced approximately 9.6 – 15.5%. The numerical results obtained is based on the size of the selected edited region.

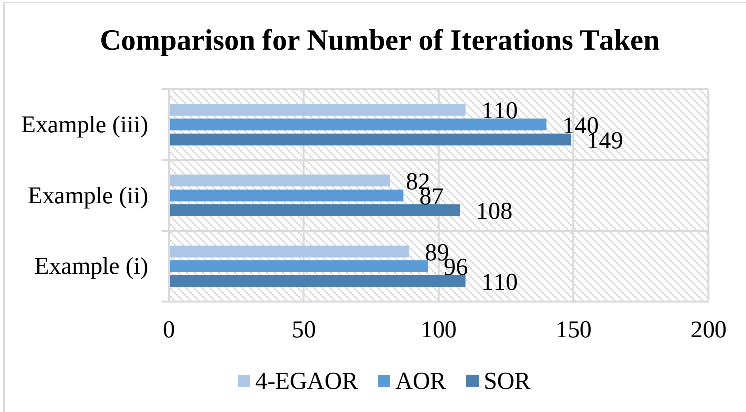


Figure 5. Number of iterations used.

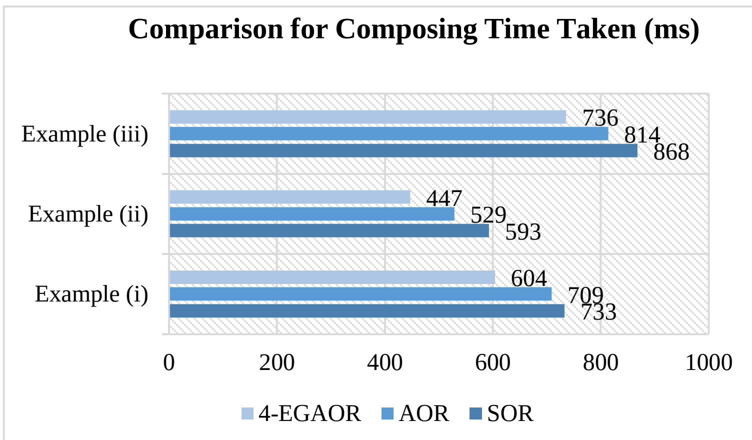


Figure 6. Composing time taken.

Subsequently, the new generated images after blending process are shown in Figures 7, 8 and 9. In order to assess the quality of the images produced, we used quantitative analysis. The quality metric used in this paper is the structural similarity index measure (SSIM) by Wang and Bovik (2002). The images produced by SOR method are used as reference image and then compared with the images produced by AOR and 4-EGAOR methods. The index values obtained are tabulated in Table 1, as follows,

Table 1. SSIM Index Values

Experiment	**Iterative Methods**	
	AOR	**4-EGAOR**
(i)	1.0000	0.9999
(ii)	1.0000	1.0000
(iii)	1.0000	1.0000

The range of SSIM index varies from -1 to 1 and value 1 indicates that both images are similar. From Table 1, all the SSIM index values obtained are close to 1 with four decimal places, which implies that the images produced by all the proposed iterative methods are very similar.

(i)

a. SOR ($\omega = 1.88$) b. AOR ($\omega = 1.86, r = 1.90$) c. 4-EGAOR ($\omega = 1.84, r = 1.92$)

Figure 7. Output images generated for Example (i) with the size of 10229 pixels.

(ii)

a. SOR ($\omega = 1.78$) b. AOR ($\omega = 1.75, r = 1.82$) c. 4-EGAOR ($\omega = 1.75, r = 1.79$)

Figure 8. Output images generated for Example (ii) with the size of 5204 pixels.

(iii)

a. SOR ($\omega = 1.90$) b. AOR ($\omega = 1.86, r = 1.91$) c. 4-EGAOR ($\omega = 1.85, r = 1.90$)

Figure 9. Output images generated for Example (iii) with the size of 6186 pixels.

CONCLUSION

In this study, both point and block iterative methods are implemented for solving Poisson image blending problem in order to examine its efficiency. From the numerical results obtained, 4-EGAOR performed the best as compared to SOR and AOR iterative methods in terms of both number of iterations and composing time. This is because the 4-

EGAOR iterative method implements the complexity reduction technique by calculating a group of 4 points simultaneously in one loop of iteration.
Thus, it performed better than the proposed point iterative methods. Finally, the new blended images generated from the three proposed iterative methods showed excellent quality as evaluated using the SSIM index.

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