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A GUARANTEED PURSUIT TIME IN A DIFFERENTIAL GAME IN HILBERT SPACE

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ABSTRACT We study a pursuit differential game problem of one pursuer and one evader in the Hilbert space l_2. The differential game is described by an infinite number of first-order 2-systems of linear differential equations. The control functions of players are subjected to integral constraints. A game is started from the given initial position z^0. It is given another point z^1 in the space l_2. If the state of the infinite system coincides with the point z^1 at some time, then pursuit is considered completed. Our purpose is to obtain an equation to find a guaranteed pursuit time and construct a strategy for the pursuer.

ABSTRAK Kami mengkaji satu permasalahan permainan pembezaan pengejaran bagi satu pengejar dan satu pengelak di dalam ruang Hilbert l_2. Permainan pembezaan diterangkan oleh bilangan tak terhingga sistem-2 persamaan pembezaan linear peringkat satu. Fungsi kawalan pemain bergantung kepada kekangan kamiran. Permainan dimulakan dari kedudukan awal z^0 yang diberi. Satu lagi titik z^1 diberi dalam ruang l_2. Jika keadaan sistem tak terhingga berkenaan bertepatan dengan titik z^1 pada suatu masa, maka pengejaran dianggap tamat. Tujuan kami adalah untuk memperolehi satu persamaan untuk mencari satu masa pengejaran yang pasti dan membina satu strategi untuk pengejar.

INTRODUCTION

Differential games were initiated by Isaacs, (1965). Differential games in finite dimensional Euclidian spaces were studied by many researchers. A large and growing body of literature has investigated two person differential games and fundamental contributions were made by researchers such as Friedman, 1971; Krasovskii & Subbotin, 1988; Lewin, 1994; Petrosyan, 1993; Pontryagin, 1988.

There are mainly two constraints on control functions of players: geometric and integral constraints. In-views of the amount of works been done in developing the differential games, the integral constraints have been extensively discussed by many researchers with various approaches.

Differential games of many players were first studied and a sufficient condition of completion of pursuit was obtained in the work of Satimov et al., (1983). In the works of Azamov & Samatov, 2000; Samatov, 2013; Π-strategy is defined for pursuer with integral constraint. The paper of Chikrii & Belousov, (2010) is devoted to a linear pursuit differential game with integral constraint. In the works of Ibragimov et al., 2011; Kuchkarov, 2013; Subbotin & Ushakov, 1975; optimal strategies of players were constructed and a formula to find the optimal pursuit time or the value of the game was found. A sufficient condition of evasion was obtained in the paper of Ibragimov & Salleh, (2012) for a differential game with coordinate-wise integral constraints. Also, the works of Huseyin & Huseyin, 2012; Konovalov, 1987; Lokshin, 1990; Nikolskii, 1969; Okhezin, 1977; Solomatin, 1984; relate to linear differential games with integral constraints where games of kind were studied.

One of the powerful methods in studying the differential game and control problems in systems with distributed parameters is the decomposition method. This method was used to study the control problems described by partial differential equations in the works of Avdonin & Ivanov, 1995; Azamov & Ruziboyez, 2013; Butkovsky, 1969; Chernous'ko, 1992.

Using the same method, differential game problems were reduced to ones described by infinite systems of differential equations in the papers of Ibragimov, 2002; Mamatov, 2008; Satimov & Tukhtasinov, 2006; Tukhtasinov & Mamatov, 2008; Tukhtasinov & Mamatov, 2009. It should be noted that in the paper of Ibragimov, (2002), optimal pursuit time was found and optimal strategies of players were constructed.

Hence, there is an important connection between the control problems described by PDE and those described by infinite systems of differential equations. Control and differential game problems described by infinite system of differential equations are of independent interest and can be investigated within one theoretical framework independently of those described by PDE.

There are many works devoted to control and differential game problems described by infinite system of differential equations. In the paper of Alias et al., (2017), an evasion differential game of infinitely many evaders from infinitely many pursuers was studied in Hilbert space l_2 and evasion strategies were constructed in explicit form. In the paper of Ibragimov, (2013), the full solution of optimal pursuit differential game problem was obtained for the infinite system of differential equations with positive coefficients in Hilbert space l_2^r. In the case of negative coefficients a control problem was studied in the paper of Ibragimov & Ja'afaru, (2011), and optimal pursuit differential game problem was studied in the paper of Ibragimov et al., (2017). Optimal strategies of inertial players were constructed in the paper of Ibragimov et al., (2015), where the control resources of pursuers need not be greater than that of evader.

In the paper Ibragimov, (2013), a differential game problem described by the infinite system

$$\begin{cases} \dot{x}_k = -\alpha_k x_k - \beta_k y_k + u_{1k} - v_{1k}, & x_k(0) = x_{k0} \\ \dot{y}_k = \beta_k x_k - \alpha_k y_k + u_{2k} - v_{2k}, & y_k(0) = y_{k0} \end{cases}, \ k = 1, 2, \cdots, \tag{1}$$

was examined in l_2, where α_k, β_k are given real numbers, $u = (u_{11}, u_{12}, u_{21}, u_{22}, \cdots)$ and $v = (v_{11}, v_{12}, v_{21}, v_{22}, \cdots)$ are control parameters of pursuer and evader, respectively, $x_0 = (x_{10}, x_{20}, \cdots) \in l_2$, $y_0 = (y_{10}, y_{20}, \cdots) \in l_2$.

The purpose of pursuer is to bring the state to the origin of the space l_2 against any action of the Evader who exactly tries to avoid this. In that paper, optimal pursuit time was found and optimal strategies of players were constructed.

In the present paper, we investigate a pursuit differential game problems described by (1) where pursuer tries to bring the state of the system from a given initial state z^0 to another given one z^1 for a finite time. We find conditions under which pursuit can be completed. Note that previous studies of differential games described by (1) have only dealt with the case $z^1 = 0$.

STATEMENT OF PROBLEM

We consider the space

$$l_2 = \left\{ \xi = (\xi_1, \xi_2, \ldots) \mid \sum_{j=1}^{\infty} |\ \xi_k\ |^2 < \infty,\ \xi_k \in \mathbb{R}^2 \right\}$$

with inner product and norm defined by

$$\langle \xi, \eta \rangle = \sum_{j=1}^{\infty} \xi_k \eta_k,\ \xi, \eta \in l_2,\ \| \xi \| = \left(\sum_{j=1}^{\infty} |\xi_k|^2 \right)^{1/2}.$$

In the space l_2, we study a differential game described by the following infinite system of 2-systems of 1st-order differential equations

$$\begin{matrix} \dot{x}_k = -\alpha_k x_k - \beta_k y_k - u_{1k} + v_{1k}, & x_k(0) = x_k^0 \\ \dot{y}_k = \beta_k x_k - \alpha_k y_k - u_{2k} + v_{2k}, & y_k(0) = y_k^0 \end{matrix}, k = 1, 2, \ldots, \tag{2}$$

with initial states of pursuer $x^0 = (x_1^0, x_2^0, \ldots) \in l_2$ and evader $y^0 = (y_1^0, y_2^0, \ldots) \in l_2$ where α_k, β_k are real numbers and $\alpha_k > 0,\ k = 1, 2, \ldots,\ u = (u_{11}, u_{12}, u_{21}, u_{22}, \ldots)$ is a control parameter of the pursuer and $v = (v_{11}, v_{12}, v_{21}, v_{22}, \ldots)$ is that of evader.

Let $x^1 = (x_1^1, x_2^1, ...) \in l_2$, $y^1 = (y_1^1, y_2^1, ...) \in l_2$ be another state, and let T be a sufficiently large positive number.

Definition 2.1 *A function* $w(\cdot)$, $w:[0,T] \to l_2$, *with measurable coordinates* $w_{1k}(t), w_{2k}(t)$, $0 \le t \le T$, $k = 1, 2, \ldots$, *so that* $w(\cdot) = (w_{11}(\cdot), w_{21}(\cdot), w_{12}(\cdot), w_{22}(\cdot), ...)$ *is subjected to*

$$\sum_{k=1}^{\infty} \int_0^T \left(w_{1k}^2(s) + w_{2k}^2(s) \right) ds \le \rho_0^2,$$

is called admissible control, where ρ_0 *is a given positive number.*

Let $S(\rho_0)$ denote the set of all admissible controls.

Definition 2.2 *Admissible controls of pursuer and evader are defined as the functions* $u(\cdot) \in S(\rho)$ *and* $v(\cdot) \in S(\sigma)$ *, respectively, where* ρ *and* σ *are given positive numbers.*

Definition 2.3 *The strategy of pursuer is defined as a function*

$$u(t,v) = (u_1(t,v), u_2(t,v), ...), \quad u:[0,T] \times l_2 \to l_2$$

whose components $u_k = (u_{1k}, u_{2k})$ *has the form*

$$u_k(t,v) = v_k(t) + w_k(t), \; w_k = (w_{k1}, w_{k2}), \; v_k = (v_{1k}, v_{2k}),$$

for which the system (2) *has a unique solution at* $u(t) = u(t,v)$, *where* $v(\cdot) = (v_1(\cdot), v_2(\cdot), ...)$ *is any admissible control of evader, and* $w(\cdot) = (w_1(\cdot), w_2(\cdot), ...) \in S(\rho - \sigma)$ *is any function.*

Denote

$$z(t) = \left(z_1(t), z_2(t), ... \right), \; z_k(t) = \left(x_k(t), y_k(t) \right), \; | z_k | = \sqrt{x_k^2 + y_k^2},$$
$$z^0 = \left(z_1^0, z_2^0, ... \right) = \left(x_1^0, y_1^0, x_2^0, y_2^0, ... \right),$$
$$z^1 = \left(z_1^1, z_2^1, ... \right) = \left(x_1^1, y_1^1, x_2^1, y_2^1, ... \right).$$

Definition 2.4 *We say that pursuit can be completed for the time* $\theta > 0$ *in the game* (2) *if for a strategy* $u(t,v)$ *of pursuer and any admissible control* $v(t)$ *of evader,* $z(\tau) = z^1$ *at some* τ, $0 \le \tau \le \theta$. *The number is also called a guaranteed pursuit time.*

Problem 2.5 *Find an equation for guaranteed pursuit time in differential game* (2)*, and construct the strategy of pursuer.*

Let

$$H_k(t)=\begin{bmatrix} e^{-\alpha_k t}\cos(\beta_k t) & -e^{-\alpha_k t}\sin(\beta_k t) \\ e^{-\alpha_k t}\sin(\beta_k t) & e^{-\alpha_k t}\cos(\beta_k t) \end{bmatrix}, k=1,2,...$$

The matrix $H_k(t)$ possesses the following properties

(i) $H_k(h+t)=H_k(t)H_k(h)=H_k(h)H_k(t),$

(ii) $|H_k(t)z_k|=|H_k^*(t)z_k|=e^{-\alpha_k t}|z_k|,$

(iii) $|H_k(t)H_k^*(t)z_k|=|H_k^*(t)H_k(t)z_k|=e^{-2\alpha_k t}|z_k|,$

where H^* is the transpose of matrix H.

SOLUTION OF A CONTROL PROBLEM

We study a control problem described by the following infinite system of differential equations

$$\begin{aligned} \dot{x}_k &= -\alpha_k x_k-\beta_k y_k+w_{1k}, \quad x_k(0)=x_k^0 \\ \dot{y}_k &= \beta_k x_k-\alpha_k y_k+w_{2k}, \quad y_k(0)=y_k^0 \end{aligned}, k=1,2,..., \tag{3}$$

where $w=(w_{11},w_{21},w_{12},w_{22},...)$ is control parameter.

Let $C(0,T;l_2)$ denote the space of continuous functions $z(\cdot)=(z_1(\cdot),z_2(\cdot),...)$ with values $z(t)\in l_2,\ 0\le t\le T$.

If $w(\cdot)\in S(\rho)$, then the infinite system of differential equations (3) has the only solution $z(t)=(z_1(t),z_2(t),...),\ 0\le t\le T$, in $C(0,T;l_2)$ (Ibragimov et al., (2008)), where

$$z_k(t)=H_k(t)\left(z_k^0+\int_0^t H_k(-s)w_k(s)ds\right),\ k=1,2,... \tag{4}$$

We study the following control problem for the infinite system of differential equations (3): find a time θ such that

$$z(0)=z^0,\ z(\theta)=z^1.$$

Let

$$E(t)=\sum_{k=1}^{\infty}\left(2\,|\,z_k^0\,|^2\; B_k(t)+2\,|\,z_k^1\,|^2\; A_k(t)\right), t>0,\; z^0, z^1\in l_2, \tag{5}$$

where

$$A_k(t)=\frac{2\alpha_k}{1-e^{-2\alpha_k t}},\, B_k(t)=\frac{2\alpha_k}{e^{2\alpha_k t}-1},\; k=1,2,...$$

The following statement can be proved similar to Lemma 1 in Ibragimov and Ja'afaru, (2011).

Lemma 3.1 *Let* $z^0, z^1\in l_2$*, and the series*

$$\sum_{k=1}^{\infty}\alpha_k\,|\,z_k^1\,|^2 \tag{6}$$

be convergent. Then, for any t > 0, the series E(t) converges.

It is not difficult to see that, for any $k=1,2,...$, both of the functions $B_k(t)$ and $A_k(t)$ are decreasing on $(0,+\infty)$, $B_k(t)\to+\infty$ and $A_k(t)\to+\infty$ as $t\to 0^+$, $B_k(t)\to 0^+$ and $A_k(t)\to 2\alpha_k$ as $t\to+\infty$.

It can be shown that the series $E(t)$ possesses the following properties.

(i) $E(t)$ is decreasing on $(0,+\infty)$;
(ii) $E(t)\to+\infty$ as $t\to 0^+$,
(iii) $E(t)\to 4\sum_{k=1}^{\infty}\alpha_k\,|\,z_k^1\,|^2$ as $t\to+\infty$.

From properties *(i)* and *(iii)*, we can see that

$$E(t)>4\sum_{k=1}^{\infty}\alpha_k\,|\,z_k^1\,|^2, t>0. \tag{7}$$

Consider the equation

$$E(t)=\rho_0^2, t>0, \tag{8}$$

and the inequality

$$\rho_0^2 > 4\sum_{k=1}^{\infty}\alpha_k \mid z_k^1 \mid^2 . \tag{9}$$

The above mentioned properties of $E(t)$ allows us to conclude that equation (8) has a root if and only if (9) is satisfied and the root is unique.

Now, we give a solution to the control problem.

Theorem 3.2 *Let* $z^0, z^1 \in l_2$ *and* (9) *be satisfied. Then there exists a control* $w(\cdot) \in S(\rho_0)$ *to transfer the state of the system* (3) *from the initial state* z^0 *to the state* z^1.

Proof: A. Define the control by

$$w_k(t) = H_k^*(-t)\left[H_k(-\theta)z_k^1 - z_k^0\right]B_k(\theta),\ k=1,2,...,\ 0 \le t \le \theta. \tag{10}$$

Show that (10) is admissible. Using the obvious inequality $|x-y|^2 \le 2|x|^2 + 2|y|^2$, properties of $H_k(t)$ and (10), we have

$$\begin{aligned}\sum_{k=1}^{\infty}\int_0^{\theta}\left|w_k(s)\right|^2 ds &= \sum_{k=1}^{\infty}\int_0^{\theta}\left|H_k^*(-s)\left[H_k(-\theta)z_k^1 - z_k^0\right]B_k(\theta)\right|^2 ds \\ &\le \sum_{k=1}^{\infty}\left(2e^{2\alpha_k\theta} \mid z_k^1 \mid^2 + 2\mid z_k^0 \mid^2\right)B_k^2(\theta)\int_0^{\theta} e^{2\alpha_k s}ds \\ &= \sum_{k=1}^{\infty}\left(2\mid z_k^0 \mid^2 B_k(\theta) + 2\mid z_k^1 \mid^2 A_k(\theta)\right) \\ &= E(\theta) = \rho_0^2.\end{aligned}$$

Thus, (10) is admissible.

B. Show that $z(\theta) = z^1$. Indeed,

$$\begin{aligned} z_k(\theta) &= H_k(\theta)\left(z_k^0 + \int_0^{\theta} H_k(-s)\left(H_k^*(-s)\left[H_k(-\theta)z_k^1 - z_k^0\right]B_k(\theta)\right)ds\right) \\ &= H_k(\theta)\left(z_k^0 + \left(H_k(-\theta)z_k^1 - z_k^0\right)B_k(\theta)\int_0^{\theta} e^{2\alpha_k s}ds\right) \\ &= H_k(\theta)z_k^0 + H_k(\theta)\left(H_k(-\theta)z_k^1 - z_k^0\right) \\ &= z_k^1.\end{aligned}$$

Thus, the system $z(t)$ can be transferred from z^0 to z^1 for the time θ. The proof of Theorem 3.2 is complete.

GUARANTEED PURSUIT TIME

Let us consider differential game (2). It is not difficult to verify that

$$z_k(t) = H_k(t)\left(z_k^0 - \int_0^t H_k(-s)u_k(s)ds + \int_0^t H_k(-s)v_k(s)ds\right). \tag{11}$$

Consider the equation

$$E(t) = (\rho - \sigma)^2,\ t > 0, \tag{12}$$

and the inequality

$$(\rho - \sigma)^2 > 4\sum_{k=1}^{\infty} \alpha_k \mid z_k^0 \mid^2 . \tag{13}$$

As mentioned above that (12) has a root $t = \theta_1$ if and only if (13) holds, and the root is unique. We can assume, by choosing T if necessary, that $\theta_1 < T$. We prove the following statement.

Theorem 4.1 *If* (13) *is satisfied and* $\rho > \sigma$ *, then* θ_1 *is guaranteed pursuit time in the game* (2).

Proof: A. We construct the pursuer's strategy on [0, θ_1] as follows

$$u_k(t,v) = v_k(t) - H_k^*(-t)\left[H_k(-\theta_1)z_k^1 - z_k^0\right]B_k(\theta_1),\ 0 \le t \le \theta_1,\ k = 1,2,... \tag{14}$$

Show that strategy (14) is admissible. Using the Minkowskii inequality and the fact that $v(\cdot)$ belongs to $S(\sigma)$, we have

$$\begin{aligned}
\left(\sum_{k=1}^{\infty}\int_0^{\theta_1}\left|u_k(s)\right|^2 ds\right)^{1/2} &= \left(\sum_{k=1}^{\infty}\int_0^{\theta_1}\left|v_k(s) - H_k^*(-s)\left[H_k(-\theta_1)z_k^1 - z_k^0\right]B_k(\theta_1)\right|^2 ds\right)^{1/2} \\
&\le \left(\sum_{k=1}^{\infty}\int_0^{\theta_1}\left|v_k(s)\right|^2 ds\right)^{1/2} \\
&+ \left(\sum_{k=1}^{\infty}\int_0^{\theta_1}\left|H_k^*(-s)\left[H_k(-\theta_1)z_k^1 - z_k^0\right]B_k(\theta_1)\right|^2 ds\right)^{1/2} \\
&\le \sigma + \left(\sum_{k=1}^{\infty}\left|H_k(-\theta_1)z_k^1 - z_k^0\right|^2 B_k^2(\theta_1)\int_0^{\theta_1} e^{2\alpha_k s} ds\right)^{1/2}.
\end{aligned} \tag{15}$$

Applying the obvious inequality $|x-y|^2 \le 2|x|^2 + 2|y|^2$ in (15), we have

$$\begin{aligned}\left(\sum_{k=1}^{\infty}\int_0^{\theta_1}|u_k(s)|^2 ds\right)^{1/2} &\le \sigma + \left(\sum_{k=1}^{\infty}\left(2|z_k^0|^2 + 2|z_k^1|^2 e^{2\alpha_k\theta_1}\right)B_k^2(\theta_1)\int_0^{\theta_1} e^{2\alpha_k s} ds\right)^{1/2} \\ &= \sigma + \left(\sum_{k=1}^{\infty}\left(2|z_k^0|^2 B_k(\theta_1) + 2|z_k^1|^2 A_k(\theta_1)\right)\right)^{1/2} \\ &= \sigma + \rho - \sigma = \rho.\end{aligned}$$

B. Show that pursuit is completed. Using (11) and (14), we obtain

$$\begin{aligned} z_k(\theta_1) &= H_k(\theta_1)\Big(z_k^0 + \int_0^{\theta_1} H_k(-s)v_k(s)ds \\ &\quad - \int_0^{\theta_1} H_k(-s)\left(v_k(s) - H_k^*(-s)\left[H_k(-\theta_1)z_k^1 - z_k^0\right]B_k(\theta_1)\right)ds\Big) \\ &= H_k(\theta_1)\left(z_k^0 + \left[H_k(-\theta_1)z_k^1 - z_k^0\right]B_k(\theta_1)\int_0^{\theta_1} e^{2\alpha_k s}ds\right) \\ &= H_k(\theta_1)z_k^0 + H_k(\theta_1)\left[H_k(-\theta_1)z_k^1 - z_k^0\right] = z_k^1.\end{aligned}$$

The proof of the theorem is complete.

CONCLUSION

In the present paper, a pursuit differential game problem described by an infinite system of 2-systems of 1st-order differential equations has been studied in Hilbert space l_2. Integral constraints on the control functions of the players are imposed.

We have solved a control problem to transfer the state of the system (3) from the given initial state z^0 to another given state z^1 for finite time. Also, we have obtained sufficient conditions for the completion of the game, we have given an equation for guaranteed pursuit time and constructed a strategy for the pursuer to complete the pursuit in the game (2).

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